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ON DETONATION OF AN EXPLOSIVE LAYER

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Results of calculating hydrodynamic parameters behind the front of a detonation wave in a plane-parallel layer of explosive in the first stage of detonation development in the wake of the front of the initiating shock wave are presented.

The problem of determination of the parameters behind the front of shock waves initiated in the environment in detonation of a plane-parallel layer of explosive (Expl) has been discussed many times [1, 2]. However, in most cases the parameters behind the front of a detonation wave (DW) and shock waves (SW) initiated by it in the environment were calculated numerically. A number of works [3–6] that seek to investigate break decay (BD) occurring in emergence of skew shock waves on the contact surfaces interfaces between different media allow a new approach to the given problem at least in those cases where detonation is stationary and the hydrodynamic theory of shock waves is applicable. According to this theory [1], the propagation of a detonation wave is caused by the movement of a shock wave in the explosive. If its amplitude is higher than a certain quantity p_d, then the shock wave is capable of exciting an intense chemical reaction behind its front: constancy of shock-wave parameters will then be maintained and stationarity of the detonation process as a whole will be ensured by the energy of this chemical reaction. A detonation wave has a more complex structure than a shock wave, but its velocity is determined by the velocity of the latter. Leaving aside the problems associated with the zone of the chemical reaction, we note that the laws of conservation of mass and momentum written for the parameters of the medium behind and in front of detonation and shock waves have the same form. Only the Hugoniot equation changes greatly because a term characterizing the chemical reaction behind the front of the initiating wave is added to it. Since in investigating of the special features of the interaction between skew shock waves and interfaces (Int) of different media [3–6] all general relations were obtained without explicit use of the Hugoniot equation, the conclusions of these studies will also be true for the case where a detonation wave emerges of the interface. This conclusion is confirmed in [2], where it is found that, from the formal point of view, the detonation process is identical to deflagration of an explosive precompressed by the shock wave. This gives grounds for dividing an analysis of the process of propagation of a detonation wave along a plane-parallel layer of explosive into two successive stages: 1) movement of the shock wave, which initiates detonation along the still "cold" explosive; 2) chemical decomposition of the explosive and separation of the detonation products. In the present work, we study just the first stage with account for the fact that the total velocity of propagation of the shock-wave front D is constant due to the energy liberated in the second stage.

Between two half-spaces (the upper half-space is denoted by the subscript H and the lower half-space by L) filled, correspondingly, with substances H and L let there be a layer of compact explosive of thickness T (Fig. 1a). As has already been mentioned, the propagation of a plane stationary detonation wave along this layer, with respect to its behavior on the upper and lower boundaries of the layer, is absolutely similar to the problem of a glancing shock wave [4]. An investigation of this problem showed [4] that at the interface along

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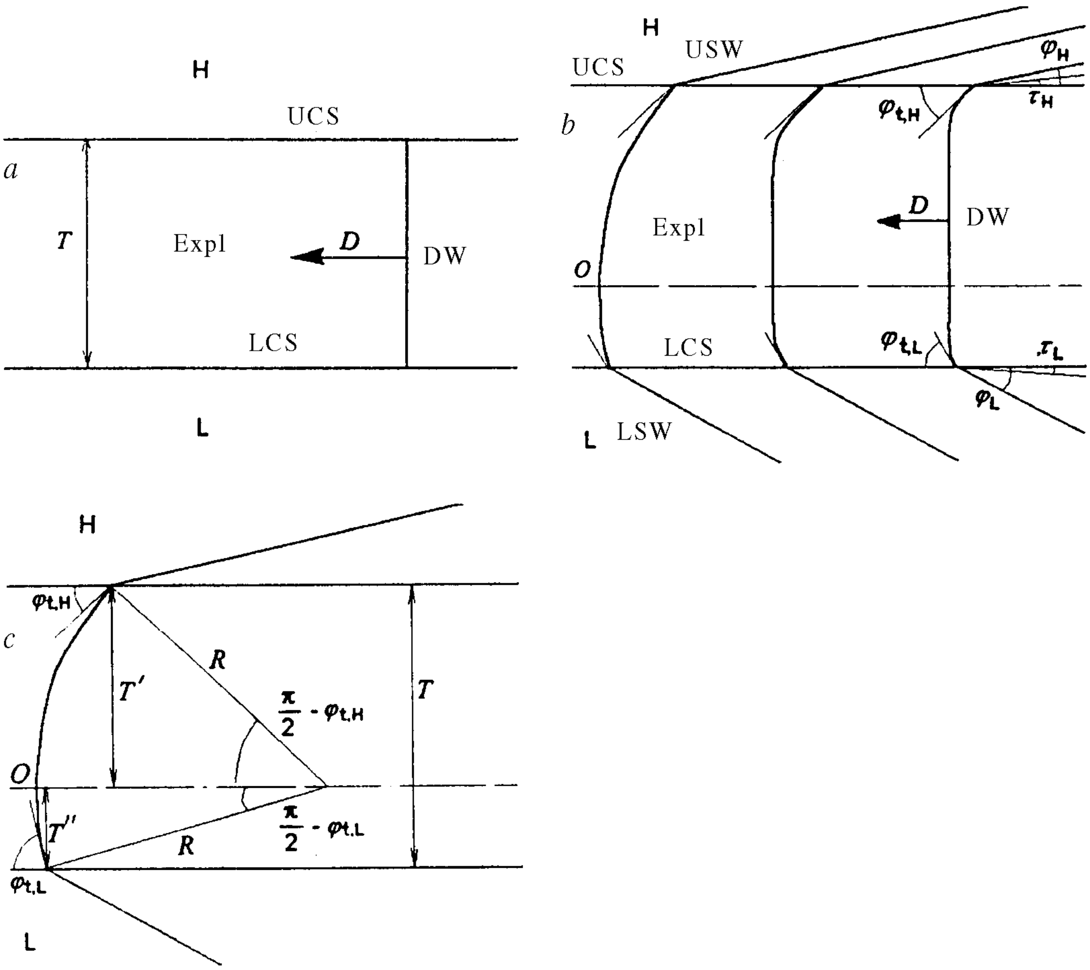

Fig. 1. Initial scheme of the problem (a), dynamics of the development of the detonation-wave shape (b), and scheme for determination of the approximating curvature of the front (c).

which the glancing shock wave propagates, there occurs break decay with formation of a curvilinear shock wave corresponding to the mode of weak shockless irregular reflection (according to the classification of [3]). The conclusions of [6] allow one to extend the above conclusion to all possible combinations of interfaces. The above relation between the detonation and shock waves gives grounds to assume that the same form of break decay must also be observed at both interfaces (UInt – the upper interface, LInt – the lower interface) in the movement of a plane detonation wave along a layer of thickness *T*. This means (Fig. 1b) that the front of the detonation wave near the upper and lower interfaces will acquire a curvilinear character, with the convex side of the curve always being directed toward the movement of the detonation wave. Plane shock waves (USW for the upper and LSW for the lower medium) originate in the media H and L at the points of contact of the detonation shock wave with the upper and lower interfaces. According to [5], the angle of contact between the detonation wave and the upper interface is

$$\varphi_{t,H} = \arcsin \sqrt{\frac{1 - \dfrac{1-m}{1-n_H}\,\alpha_H}{1-\alpha_H^2}}\,, \qquad (1)$$

and the relation between $\varphi_{t,H}$ and the angle φ_H formed by the upper shock wave and the horizonal (Fig. 1b) is

$$\sin\varphi_H = \sin\varphi_{t,H}\sqrt{\alpha_H\frac{1-n_H}{1-m}} = \sqrt{\alpha_H\frac{\frac{1-n_Í}{1-m}-\alpha_H}{1-\alpha_H^2}}\,, \tag{2}$$

where $n_H = \rho_0/\rho$ (near the upper interface), $m = \rho_{0,H}/\rho_H$, and $\alpha_H = \rho_0/\rho_{0,H}$; ρ_0 and $\rho_{0,H}$ are the initial densities of the explosive and substance H, and ρ and ρ_H are the densities behind the front of the detonation wave and the upper shock wave, respectively. The same relations can be written for the lower interface:

$$\varphi_{t,L} = \arcsin\sqrt{\frac{1-\frac{1-\kappa}{1-n_L}-\alpha_L}{1-\alpha_L^2}}\,, \tag{3}$$

$$\sin\varphi_L = \sqrt{\alpha_L\frac{\frac{1-n_L}{1-\kappa}-\alpha_L}{1-\alpha_L^2}}\,, \tag{4}$$

where $\kappa = \rho_{0,L}/\rho_L$, $\alpha_L = \rho_0/\rho_{0,L}$, and $n_L = \rho_0/\rho$ (near the lower interface); $n_L \neq n_H$ except for cases where substances H and L are alike and $\varphi_{t,H} = \varphi_{t,L}$, $\varphi_H = \varphi_L$, $\alpha_H = \alpha_L$, and κ = m.

We calculate the values of the parameters behind the front of the shock wave that initiates detonation from the formulas

$$p - p_0 = \rho_0 D^2\sin^2\varphi\,(1-n)\,,\quad q = D\cot\varphi\sqrt{1+\tan^2\varphi n^2}\,,$$

$$\vartheta = \varphi - \arccos\frac{1}{\sqrt{1+\tan^2\varphi n^2}}\,,\quad p - p_0 = \frac{a^2\rho_0\,(n^{-1}-1)}{n\,[1-(s-1)\,(n^{-1}-1)]^2} = \frac{a^2\rho_0\,(n-1)}{[1-s\,(1-n)]^2}\,, \tag{5}$$

where s and a are the parameters of the shock adiabat of the "cold" explosive

$$D = a + sU\,, \tag{6}$$

and the angle φ is formed by the tangent to the surface of the shock-wave front and the horizonal, $n = n(\varphi)$. The pressures on both sides of the upper and lower interfaces must be the same. This makes it possible to find m and κ. Let $p = h(m) + p_0$ and $p = l(\kappa) + p_0$ be the equations of state of medium H and L, respectively. The pressure behind the front of the initiating shock wave in the vicinity of the upper contact point is

$$p_{t,H} = p_0 + \rho_0\,D^2\sin^2\varphi_{t,H}\,(1-n_H)\,, \tag{7}$$

where $n_H = n(\varphi_{t,H})$. Using (1) and allowing for the fact that

$$D \sin \varphi_{t,H} = D \sqrt{\frac{1 - \dfrac{1-m}{1-n_H}\alpha_H}{1-\alpha_H^2}}\ , \tag{8}$$

and $p_{t,H} = h(m) + p_0$, we write for medium H

$$h\,(m) = \rho_0 D^2 \frac{1 - n_H - (1-m)\,\alpha_H}{1-\alpha_H^2}\,. \tag{9}$$

Having performed quite similar operations for the lower interface, we obtain

$$l\,(\kappa) = \rho_0 D^2 \frac{1 - n_L - (1-\kappa)\,\alpha_L}{1-\alpha_L^2}\,, \tag{10}$$

where $n_L = n(\varphi_{t,L})$ and $\varphi_{t,L}$ is the angle formed by the detonation wave and the lower interface. We transform Eqs. (8) and (9) to

$$n_H = 1 - (1-m)\,\alpha_H - (1-\alpha_H^2)\,\frac{h\,(m)}{\rho_0 D^2} \tag{11}$$

and

$$n_L = 1 - (1-\kappa)\,\alpha_L - (1-\alpha_L^2)\,\frac{l\,(\kappa)}{\rho_0 D^2}\,, \tag{12}$$

respectively. Having substituted successively (11) and (12) into the fourth equation of (5), we obtain two transcendental equalities in m and κ. Inverse substitution of the calculated m and κ into (11) and (12) yields n_H and n_L, which allows determination of the angles $\varphi_{t,H}$, φ_H, $\varphi_{t,L}$, and φ_L using (1)–(4). This, in turn, specifies the velocities of propagation of the upper shock wave ($D_H = D \sin \varphi_H$) and the lower shock wave ($D_L = D \sin \varphi_L$); then we can calculate all hydrodynamic parameters of flows originating behind their fronts.

The rate of change of the parameters of the flow behind the detonation-wave front is completely determined by the rate of bending of the plane detonation wave as it approaches the corresponding interface. The step-by-step dynamics of the establishment of a stable stationary configuration of the detonation wave in a plane-parallel layer is shown in Fig. 1b. It is clear that the process may be assumed to be completely steady-state only when the initially plane detonation wave between two bent Mach waves near the upper and lower interfaces degenerates to a point (O in Fig. 1a and b), at which the tangent to the detonation-wave front is perpendicular to the initial velocity D. If in a first approximation we approximate the Mach waves by the corresponding circular arcs with the same radius (a consequence of the same initial intensity of the detonation wave at all points of the front), then the shape of the wave formed and the rate of change of the parameters of the flow behind the front can be found exactly. The radius of curvature of the stationary detonation wave formed is

$$R = \frac{T}{\sin\left(\dfrac{\pi}{2} - \varphi_{t,H}\right) + \sin\left(\dfrac{\pi}{2} - \varphi_{t,L}\right)} = \frac{T}{\cos \varphi_{t,H} + \cos \varphi_{t,L}}\,. \tag{13}$$

In this case, the available data are sufficient to determine the parameters of the flow in all three media with an accuracy specified by the approximation chosen. We have already calculated the densities near the upper and lower interfaces (n_H, n_L, m, and κ); the pressure can be calculated by formulas (9) and (10) in the regions H and L, respectively. In the region occupied by the explosive

$$p_H = p_0 + \rho_0 D^2 \sin^2 \varphi \,(1 - n\,(\varphi))\,, \tag{14}$$

$$n\,(\varphi) = \frac{s-1}{s} + \frac{a}{sD \sin \varphi}\,, \tag{15}$$

where φ changes from the upper to the lower interface along the circular arc, first increasing from $\varphi_{t,H}$ to $\pi/2$ at point O and then decreasing to $\varphi_{t,L}$ at the lower interface. The value of n is less than zero, because, as a rule, $s > 1$ and $n < 1$ in cases where $a < D \sin \varphi$. If we adopt a length scale perpendicular to both interfaces (Fig. 1c), having brought its origin into coincidence with the upper interface, then instead of φ we can use the linear characteristic

$$\frac{x}{R} = \cos \varphi_{t,H} + \begin{cases} -\cos \varphi\,, & \varphi_{t,H} \le \varphi \le \dfrac{\pi}{2}\,, \\ \cos \varphi\,, & \dfrac{\pi}{2} \le \varphi \le \varphi_{t,L}\,, \end{cases} \tag{16}$$

which is much convenient for practical calculations. The inverse dependence turns out to be even simpler:

$$\varphi = \arccos \left(\left| \cos \varphi_{t,H} - \frac{x}{R} \right| \right), \quad 0 \le x \le T\,. \tag{17}$$

In this case,

$$p = p_0 + \frac{\rho_0 D^2}{s} \sin^2 \varphi \left(1 - \frac{a}{D \sin \varphi} \right) \tag{18}$$

holds true for all $0 \le x \le T$ rather than (14). The total velocity of the flow behind the upper shock wave, detonation wave, and lower shock wave is determined by the formulas

$$q_H = D \cos \varphi_H \sqrt{1 + \tan^2 \varphi_H m^2}\,, \tag{19}$$

$$q = D \left| \cos \varphi_{t,H} - \frac{x}{R} \right| \left(1 + n^2 \left(\frac{1}{\left(\cos \varphi_{t,H} - \dfrac{x}{R} \right)^2} - 1 \right) \right)^{1/2} \tag{20}$$

and

$$q_L = D \cos \varphi_L \sqrt{1 + \tan^2 \varphi_L \kappa^2}\,, \tag{21}$$

and the flow bend in passage through the front is determined by

$$\vartheta_H = \varphi_H - \arccos \sqrt{\frac{1}{1 + \tan^2 \varphi_H m^2}}\,, \tag{22}$$

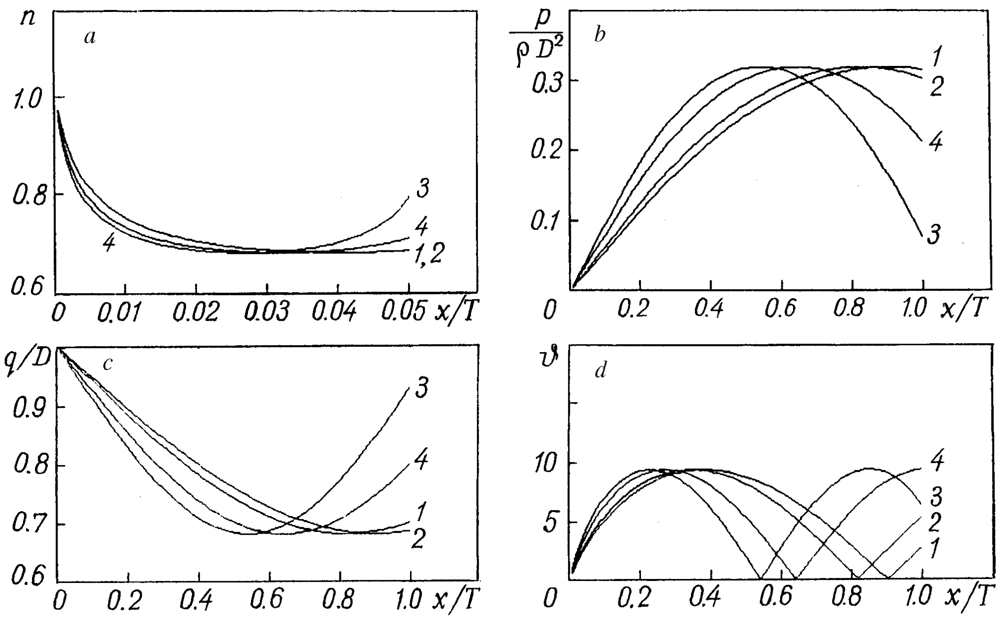

Fig. 2. Change in the parameters behind the front of the initiating wave for tungsten (1), iron (2), KCl (3), and water (4): a) $n(x)$; b) $p(x)$; c) $q(x)$; d) $\vartheta(x)$.

$$\vartheta = \arccos\left|\cos\varphi_{t,H} - \frac{x}{R}\right| - \arccos\left(\frac{1}{1 + n^2\left(\dfrac{1}{\left(\cos\varphi_{t,H} - \dfrac{x}{R}\right)^2} - 1\right)}\right)^{1/2}, \tag{23}$$

$$\vartheta_L = \varphi_L - \arccos\sqrt{\frac{1}{1 + \tan^2\varphi_L \kappa^2}} \tag{24}$$

respectively. The bend of the upper and lower interfaces in the wake of the contact points is specified by the angles (Fig. 1b)

$$\tau_H = \arccos\frac{q_H \sin\vartheta_H}{D} \tag{25}$$

$$\tau_L = \arccos\frac{q_L \sin\vartheta_L}{D} \tag{26}$$

Figure 2 presents results of calculating the hydrodynamic parameters of the media in passage of the detonation wave over a layer of trotyl (ρ_0 = 1590 kg/m^3, D = 6940 m/sec, a = 2390 m/sec, and s = 2.05 [1]) in contact with air from above and with different dense materials from below (the properties of the materials were determined from the corresponding D–U adiabatic curves) [1]. Figure 3 allows one to estimate the dynamics of the change in the degree of compression behind the shock wave in dense media and explosives and the level of pressure that develops on the contact surface as a function of α_L. As is seen, in the vicinity of the point α_L = 1 all the parameters have a very narrow extremum; however, in this case, a certain instability

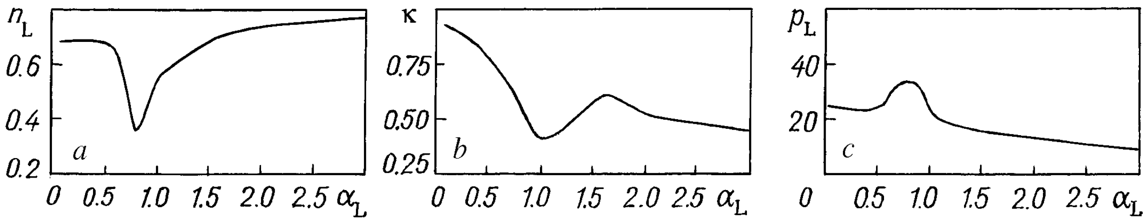

Fig. 3. Dependences $n_L(\alpha_L)$ (a), $\kappa(\alpha_L)$ (b), and $p_L(\alpha_L)$ (c). p, GPa.

TABLE 1. Some Parameters That Are Typical of the Interaction between the Shock Wave That Initiates Detonation and the Interface of the Explosive (Trotyl) with Substances of Different Density. The Layer Thickness Is $T = 0.05$ m; in All Cases the Upper Half-Space Is Filled with Air

Pair of substances	α_L	$\varphi_{t,L}$, deg	φ_L, deg	R/T	ϑ_L, deg	q_L, m/sec	p_L, GPa
Trotyl–tungsten	0.083	84.2	36.1	0.959	2.0	6762	24.17
Trotyl–iron	0.203	77.9	43.5	0.870	3.9	6535	23.15
Trotyl–KCl	0.803	36.7	30.9	0.574	25.3	5982	21.05
Trotyl–water	1.590	58.3	68.7	0.682	11.0	4712	16.64
Trotyl–alcohol	2.013	59.4	73.6	0.685	10.8	4503	16.05
Trotyl–air	1233	19.8	89.9	0.500	12.4	72.31	0.062

of the calculations was noted that could reflect on the results. Data on the characteristic angles made by the detonation wave with the interface for different pairs of substances are given in Table 1. It should be noted that for interfaces of the condensed explosive–gas type, the pressure behind the front of the initiating shock wave is relatively low (the wave turned out to be weak in all the cases) and is 0.04–0.08 GPa. Since for the overwhelming majority of industrial explosives the pressure of shock initiation is $p_d \geq 0.1$ GPa [1], we can assume that a portion of the explosive that is in contact with the gas is not initiated but is just set in motion by the shock wave; in practical explosions, it can be found in undecomposed form near the site of blasting. We can find the fraction of this substance in each specific case, specifying, for example, the experimental value of the initiating pressure and searching for x at which $p(x) \leq p_d$.

NOTATION

q, total velocity of the substance in a coordinate system tied to the detonation wave; p, pressure; n, ratio of the initial density of the explosive to the density behind the front of the initiating shock wave; m and κ, ratios of the initial density of medium H and medium L to the density behind the shock wave front; U, component of the velocity of the flow perpendicular to the shock-wave front; ρ, density; α, ratio of the initial density of the explosive to the density of the substance that is in contact with it; φ, angle between the tangent to the front of the initiating shock wave and the horizonal; D, detonation rate; T, thickness of the explosive layer; R, radius of curvature of the wave; x, distance from any point of the explosive layer to the upper boundary; $h(m)$ and $l(\kappa)$, functions of state of the "upper" and "lower" substances; ϑ, angle of rotation of the vector of the total velocity of the flow from the horizontal direction behind the shock-wave front; τ, angle of rotation of the interface behind the point of contact with the detonation-wave front. Subscripts: t, parameters for the explosive layer adjacent to any interface; H, upper half-space; L, lower half-space; d, parameters of initiation of the detonation wave; 0, initial values.

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